ACTUARIAL SOCIETY OF SOUTH AFRICA

New econometric methods for estimating risk and time preferences based on lottery-choice experiments

Andre Hofmeyr & Don Ross

Research Unit in Behavioural Economics and Neuroeconomics (RUBEN)

School of Economics, University of Cape Town

Outline

1. The importance of eliciting risk and time preferences
2. Methods for eliciting risk and time preferences
3. The statistical approach to estimating risk and time preferences
4. Risk and time preference results
5. Conclusion

The importance of eliciting risk and time preferences

- Risk and time preferences are basic inputs to estimating demand for any product – and are especially obvious components of demand for insurance.
- Economists have sometimes assumed, as a first principle, that everyone's risk preferences just reflect *objective* risk, and that people discount monetary values exponentially, so they're the inverse of interest rates. Mountains of evidence show that these assumptions seldom apply.

Heterogeneity of risk preferences

Our methods of measuring risk and time preferences have been applied to many populations, rich and poor, throughout the world. It is clear from this that different sub-groups within populations have differently *structured* preferences with respect to time, and different attitudes to risk. Our methods identify the factors that separate such sub-populations.

More specifically …

- Most people are moderately risk *averse* – willing to pay more than the actuarial cost of a risk to reduce it. But some are risk *neutral* and a few are risk *loving*. Risk loving people will pay costs to take on extra risk. Also, some people's risk preferences change at different levels of background wealth.
- Most people, in most decisions, apply consistent rates of discounting across all future intervals. But most people also discount *some* comparative values *hyperbolically*, meaning that rates get steeper as payoffs get closer in time.

Microinsurance

Failure to experimentally measure heterogeneous risk and time preferences is an important part of the reason why microinsurance take-up rates are famously low and profits from such policies are often elusive. The poor often have non-monetized forms of insurance against some specific kinds of risk, and won't pay for monetized substitutes. Our experiments can detect the influence of, and quantify the value of, factors that sub-populations treat as insurance substitutes.

While there is a burgeoning literature which applies these techniques to the design of microinsurance policies, amongst other things, this talk is primarily methodological and is intended to stimulate interest in the technologies that are available for eliciting and estimating these preferences.

Methods to elicit risk and time preferences

ACTUARIAL SOCIETY OF SOUTH AFRICA

- There are a number of different methods that can be used to elicit risk and time preferences.
- For the purposes of explanation we will discuss the Multiple Price List (MPL) which was popularised by Holt and Laury (2002) and Coller and Williams (1999) to elicit risk and time preferences, respectively.
- After discussing the basic logic embodied in an MPL we will briefly describe other methods for eliciting these preferences.

Multiple Price List (MPL) for risk preferences

- An example risk preference MPL is shown below:

Decision	Option A	Option B	Your Choice (Circle A or B)
1	R250 if dice is 1 R150 if dice is 2 3 4 5 6 7 8 9 0	R400 if dice is 1 R40 if dice is 2 3 4 5 6 7 8 9 0	A B
2	R250 if dice is 1 2 R150 if dice is 3 4 5 6 7 8 9 0	R400 if dice is 1 2 R40 if dice is 3 4 5 6 7 8 9 0	A B
3	R250 if dice is 1 2 3 R150 if dice is 4 5 6 7 8 9 0	R400 if dice is 1 2 3 R40 if dice is 4 5 6 7 8 9 0	A B
4	R250 if dice is 1 2 3 4 R150 if dice is 5 6 7 8 9 0	R400 if dice is 1 2 3 4 R40 if dice is 5 6 7 8 9 0	A B
5	R250 if dice is 1 2 3 4 5 R150 if dice is 6 7 8 9 0	R400 if dice is 1 2 3 4 5 R40 if dice is 6 7 8 9 0	A B
6	R250 if dice is 1 2 3 4 5 6 R150 if dice is 7 8 9 0	R400 if dice is 1 2 3 4 5 6 R40 if dice is 7 8 9 0	A B
7	R250 if dice is 1 2 3 4 5 6 7 R150 if dice is 8 9 0	R400 if dice is 1 2 3 4 5 6 7 R40 if dice is 8 9 0	A B
8	R250 if dice is 1 2 3 4 5 6 7 8 R150 if dice is 9 0	R400 if dice is 1 2 3 4 5 6 7 8 R40 if dice is 9 0	A B
9	R250 if dice is 1 2 3 4 5 6 7 8 9 R150 if dice is 0	R400 if dice is 1 2 3 4 5 6 7 8 9 R40 if dice is 0	A B
10	R250 if dice is 1 2 3 4 5 6 7 8 9 0	R400 if dice is 1 2 3 4 5 6 7 8 9 0	A B

The basic logic of the risk preference MPL is that you are presenting subjects with a choice between two options on every row of the table, one of which is risky and one of which is safe.

Multiple Price List (MPL) for risk preferences

- Let's look at Row 1 together.
- Note that the circles next to the amounts of money represent the numbers on a ten-sided dice.
- The numbers on the ten-sided dice are from 0 - 9 and we will use 0 to represent 10.
- Looking at Option A, you need to roll 1 on the ten-sided dice to receive R250.
- If you roll any other number from 2 - 10, you will receive R150.
- What this means is that you have a 10% chance of receiving R250 and a 90% chance of receiving R150.

Decision	Option A	Option B	Your Choice (Circle A or B)
1	R250 if dice is 1 R150 if dice is 2 3 4 5 6 7 8 9 0	R400 if dice is 1 R40 if dice is 2 3 4 5 6 7 8 9 0	A B

Multiple Price List (MPL) for risk preferences

- Looking at Option B, you need to roll 1 on the ten-sided dice to receive R400 but if you roll a number from 2 - 10 you will receive R40.
- This means that you have a 10% chance of receiving R400 and a 90% chance of receiving R40.
- Once you have made your decision between Option A and Option B, you will record this choice in the final column of the table labeled "Your Choice".

Decision	Option A	Option B	Your Choice (Circle A or B)
1	R250 if dice is 1 R150 if dice is 2 3 4 5 6 7 8 9 0	R400 if dice is 1 R40 if dice is 2 3 4 5 6 7 8 9 0	A B

Multiple Price List (MPL) for risk preferences

- Notice that as you move down the table from Row 1 to Row 10, the chance of receiving the larger amount of money under both options increases.
- Looking at Option A on Row 2, you can now roll 1 or 2 on the ten-sided dice to receive R250. On Row 1, you could only roll 1 to receive R250.
- Under Option A on Row 1 you have a 10% chance of receiving R250 but on Row 2 you have a 20% chance of receiving R250.
- Similarly for Option B on Row 2; you can roll a 1 or 2 to receive R400. On Row 1, you could only roll a 1 to receive R400.
- So, under Option B on Row 1, you have a 10% chance of receiving R400 but on Row 2 you have a 20% chance of receiving R400.

Decision	Option A	Option B	Your Choice (Circle A or B)
1	R250 if dice is 1 R150 if dice is 2 3 4 5 6 7 8 9 0	R400 if dice is 1 R40 if dice is 2 3 4 5 6 7 8 9 0	A B
2	R250 if dice is 1 2 R150 if dice is 3 4 5 6 7 8 9 0	R400 if dice is 1 2 R40 if dice is 3 4 5 6 7 8 9 0	A B

Multiple Price List (MPL) for risk preferences

- Once a person has completed the risk preference MPL you can use their choice data to determine their attitude towards risk.
- Recall that the basic logic of an MPL is that you are presenting subjects with a choice between two options on each row, one of which is safe and one of which is risky.
- In the table below, which is the safe option and which is the risky option?

Decision	Option A	Option B	Your Choice (Circle A or B)
1	R250 if dice is 1 R150 if dice is 2 3 4 5 6 7 8 9 0	R400 if dice is 1 R40 if dice is 2 3 4 5 6 7 8 9 0	A B
2	R250 if dice is 1 2 R150 if dice is 3 4 5 6 7 8 9 0	R400 if dice is 1 2 R40 if dice is 3 4 5 6 7 8 9 0	A B

Multiple Price List (MPL) for risk preferences

- Let's now add a bit more detail to the table we just looked at:

Row	Option A					Option B					Difference
	p	Rands	1-p	Rands	**EVA**	q	Rands	1-q	Rands	**EVB**	**EVA - EVB**
1	0,1	250	0,9	150	**160**	0,1	400	0,9	40	**76**	84
2	0,2	250	0,8	150	**170**	0,2	400	0,8	40	**112**	58
3	0,3	250	0,7	150	**180**	0,3	400	0,7	40	**148**	32
4	0,4	250	0,6	150	**190**	0,4	400	0,6	40	**184**	6
5	0,5	250	0,5	150	**200**	0,5	400	0,5	40	**220**	-20
6	0,6	250	0,4	150	**210**	0,6	400	0,4	40	**256**	-46
7	0,7	250	0,3	150	**220**	0,7	400	0,3	40	**292**	-72
8	0,8	250	0,2	150	**230**	0,8	400	0,2	40	**328**	-98
9	0,9	250	0,1	150	**240**	0,9	400	0,1	40	**364**	-124
10	1	250	0	150	**250**	1	400	0	40	**400**	-150

Multiple Price List (MPL) for risk preferences

- So as we just saw, the row on which an individual switches from choosing Option A to Option B reflects their attitude towards risk.
- If a person switches to Option B before row 5 then they are risk loving.
- If they switch to Option B on row 5 then they are risk neutral.
- And if they switch to Option B after row 5 then they are risk averse.
- If we adopt a simple parametric specification of an individual's utility function (specifically the Power function $U(x) = x^r$) then the row on which a person switches implies values of the Power function parameter *r*.
- Let's see what this means in our risk preference MPL.

Multiple Price List (MPL) for risk preferences

- Under expected utility theory, risk attitudes are determined by the shape of the utility function.
- A concave utility function implies risk aversion, a linear utility function implies risk neutrality and a convex utility function implies risk loving behaviour.
- Thus, if someone switches to Option B before row 5, the Power function parameter $r > 1$ (i.e. risk loving).
- If someone switches to Option B on row 5 then $r = 1$ (i.e. risk neutral).
- And if they switch after row 5 then $r < 1$ (i.e. risk averse).
- Later we'll discuss ways in which to derive more sophisticated inferences from these data.
- Let's now look at a MPL which can be used to elicit time preferences.

Multiple Price List (MPL) for time preferences

ACTUARIAL SOCIETY OF SOUTH AFRICA

- An example time preference MPL is shown below:

Decision	Option A (Pays amount below today)		Option B (Pays amount below in 1 month)	Your Choice (Circle A or B)
1	R250.00 Today	+ 10% interest =	R252.09 1 month	A B
2	R250.00 Today	+ 20% interest =	R254.20 1 month	A B
3	R250.00 Today	+ 30% interest =	R256.33 1 month	A B
4	R250.00 Today	+ 40% interest =	R258.47 1 month	A B
5	R250.00 Today	+ 50% interest =	R260.63 1 month	A B
6	R250.00 Today	+ 60% interest =	R262.81 1 month	A B
7	R250.00 Today	+ 70% interest =	R265.00 1 month	A B
8	R250.00 Today	+ 80% interest =	R267.22 1 month	A B
9	R250.00 Today	+ 90% interest =	R269.45 1 month	A B
10	R250.00 Today	+ 100% interest =	R271.70 1 month	A B

The basic logic of the time preference MPL is that you are presenting subjects with a choice between two options on every row of the table and the point at which an individual switches from A to B defines his discount rate.

Multiple Price List (MPL) for time preferences

- As you can see in the table below, the time horizon between Option A and Option B is 1 month.
- The row where an individual switches from A to B defines his discount rate.
- If the subject switches to B on row 1 then his discount rate is between 0% and 10%
- If the subject switches to B on row 2 then his discount rate is between 10% and 20%.

Decision	Option A (Pays amount below today)	Option B (Pays amount below in 1 month)	Your Choice (Circle A or B)
1	**R250.00** *+ 10% interest =* *Today*	**R252.09** *1 month*	A B
2	**R250.00** *+ 20% interest =* *Today*	**R254.20** *1 month*	A B

Multiple Price List (MPL) for time preferences

ACTUARIAL SOCIETY OF SOUTH AFRICA

- Note that by using multiple MPLs with different time horizons you can pin down an individual's discount rate for these different horizons
- Using this information you can then infer an individual's average discount rate for all time horizons.
- In the table below, we use a 3 month time horizon.

Decision	Option A (Pays amount below today)	Option B (Pays amount below in 3 months)	Your Choice (Circle A or B)
1	**R250.00** *+ 10% interest =* *Today*	**R256.33** *3 months*	A B
2	**R250.00** *+ 20% interest =* *Today*	**R262.81** *3 months*	A B

Other methods for eliciting risk and time preferences

- The MPL is a simple and effective method for eliciting risk and time preferences.
- However, other methods exist for eliciting risk and time preferences which may be preferable in particular settings.
- It is often worthwhile to use computer software to elicit these preferences because you can increase the range and complexity of the questions that are asked and thereby derive more nuanced estimates.
- Without dwelling on the details, the following slides include screenshots of the software that we used to elicit risk and time preferences.
- We have a mobile laboratory that allows us to take this software out to research subjects “in the field”.

Software for risk preferences

Software for time preferences

ACTUARIAL SOCIETY OF SOUTH AFRICA

The statistical approach to estimating risk and time preferences

- Now that we know how to elicit risk and time preferences, the next thing we need to discuss is how to estimate these preferences parametrically.
- The approach we adopt is direct estimation by maximum likelihood of a structural model of a latent choice process in which the core parameters defining risk attitudes and time preference behaviour can be estimated.
- We focus on the basic logic for estimating risk attitudes, and discuss the extension to discounting behaviour (i.e. time preferences).

Maximum likelihood estimation (MLE) of risk preferences

- Assume for the moment that utility of income is defined by a Power utility function which displays constant relative risk aversion (CRRA):

  $$U(x) = x^r$$

  where x is the lottery prize and $r \neq 0$ is a parameter to be estimated. For $r = 0$ assume $U(x) = \ln(x)$ if needed.

- Let there be two possible outcomes in a lottery. Under expected utility (EU) theory the probabilities for each outcome x_j, $p(x_j)$, are those that are used in the experimental task, so expected utility is simply the probability weighted utility of each outcome in each lottery i:

  $$EU_i = \sum_{j=1,2} [\, p(x_j) \times U(x_j) \,].$$

Maximum likelihood estimation (MLE) of risk preferences

- The expected utility (EU) for each lottery pair is calculated for a candidate estimate of r, and the index

$$\nabla EU = EU_B - EU_A$$

calculated, where EU_A is the Option A lottery and EU_B is the Option B lottery as presented to subjects.

- This latent index, based on latent preferences, is then linked to observed choices using a standard cumulative normal distribution function $\Phi(\nabla EU)$.

- This "probit" function takes any argument between $\pm\infty$ and transforms it into a number between 0 and 1. Thus we have the probit link function:

$$\text{prob(choose lottery B)} = \Phi(\nabla EU)$$

Maximum likelihood estimation (MLE) of risk preferences

- Thus the likelihood of the observed responses, conditional on the EU and Power utility specifications being true, depends on the estimates of r given the above statistical specification and the observed choices.

- The conditional log-likelihood for the model is:

$$\ln L(r; y, \mathbf{X}) = \sum_i [(\ln \Phi(\nabla EU) \times \mathbf{I}(y_i = 1)) + (\ln (1-\Phi(\nabla EU)) \times \mathbf{I}(y_i = 0))]$$

  where $\mathbf{I}(\cdot)$ is the indicator function, y_i =1(0) denotes the choice of the Option B (A) lottery in risk aversion task i, and **X** is a vector of individual characteristics.

- The parameter r is defined as a linear function of the characteristics in vector **X** which can include things like age, gender and population group.

Maximum likelihood estimation (MLE) of time preferences

- Focussing now on time discounting, assume that EU theory holds for choices over risky alternatives and that discounting is exponential.
- A subject is indifferent between two income options x_t and $x_{t+\tau}$ if and only if:

  $$U(x_t) = (1/(1+\delta)^{\tau})\ U(x_{t+\tau})$$

  where $U(x_t)$ is the utility of monetary outcome x_t for delivery at time t, δ is the discount rate, τ is the horizon for delivery of the later monetary outcome at time t+τ, and the utility function U is separable and stationary over time.
- Note that this is an indifference condition and δ is the discount rate that equalizes the present value of the *utility* of the two monetary outcomes x_t and $x_{t+\tau}$.

Maximum likelihood estimation (MLE) of time preferences

$$U(x_t) = (1/(1+\delta)^\tau)\, U(x_{t+\tau})$$

- An important thing to recognize about this specification is that it depends on estimates of the Power utility function introduced earlier.
- What this means is that one cannot infer the level of the individual discount rate without knowing or assuming something about a person's utility function.
- This identification problem implies that discount rates cannot be estimated based on discount rate experiments with choices defined solely over time-dated money flows, and that separate tasks to identify the extent of diminishing marginal utility (i.e. risk attitudes) must also be implemented.
- Thus, one must jointly estimate the parameters defining risk attitudes and time discounting (Andersen et al., 2008).

Maximum likelihood estimation (MLE) of time preferences

To illustrate this point note that the discounted utility of Option A is given by:

$$PV_A = (x_A)^r$$

and the discounted utility of Option B is:

$$PV_B = (1/(1+\delta)^\tau)\,(x_B)^r$$

where x_A and x_B are the monetary amounts in the discounting tasks presented to subjects and the utility function is assumed to be stationary over time.

- An index of the difference between these present values, conditional on r and δ, can then be defined as

$$\nabla PV = (PV_B - PV_A)$$

Maximum likelihood estimation (MLE) of time preferences

- Thus the likelihood of the discount rate responses, conditional on the EU, Power utility and exponential discounting specifications being true, depend on the estimates of r and δ, given the observed choices of subjects in the experiment.

- The conditional log-likelihood for the model is:

$$\ln L(r, \delta; y, \mathbf{X}) = \sum_i [(\ln \Phi(\nabla PV) \times \mathbf{I}(y_i=1)) + (\ln (1-\Phi(\nabla PV)) \times \mathbf{I}(y_i=0))]$$

where y_i =1(0) again denotes the choice of Option B (A) in discount rate task i, and **X** is a vector of individual characteristics.

Maximum likelihood estimation (MLE) of time preferences

- The joint likelihood of the risk aversion and discount rate responses can then be written as

  $$\ln L\ (r, \delta, \mu, \eta; y, \mathbf{X}) = \ln L^{RA} + \ln L^{DR}$$

  where L^{RA} is the log likelihood for the risk preference task and L^{DR} is the log likelihood for the time preference task.

- This expression can then be maximized using standard numerical methods to estimate the Power utility function parameter r and the exponential discount rate δ.

- Note that the estimates of r and δ maximise the likelihood of observing the data and thus best characterise it.

Risk and time preference results

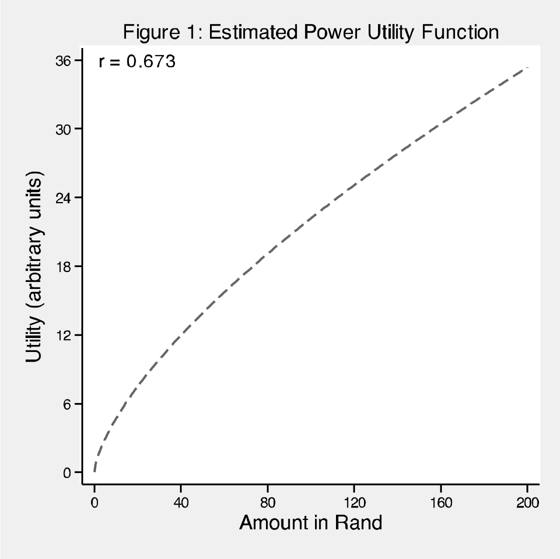
- The following results are drawn from a set of studies, which the presenters have been involved with, conducted on students at UCT and they highlight the value of experimentally eliciting and estimating risk and time preferences.
- These results are by no means exhaustive and simply provide a flavour of the type of results that can be estimated using experimental data on risk attitudes and discounting behaviour.
- Table I and Figure I on the next slide show estimates of the Power function parameter *r* from an experimental study conducted on real subjects using our experimental methodology.
- Note that the results on the next slide pool choices in the risk preference task over all subjects and thereby assume homogenous preferences.

Risk preference results: Homogenous preferences

TABLE I: EXPECTED UTILITY THEORY
MLE ESTIMATES - HOMOGENOUS PREFERENCES

	Model
	CU error Normal CDF
Power function coefficient (r)	0.673***
	(0.080)
Error	0.176***
	(0.016)
N	2640
log-likelihood	-1229.877

Results account for clustering at the individual level
Standard errors in parentheses
* p<0.10, ** p<0.05, *** p<0.01



- As is evident, the estimate of *r* = 0.673 implies a moderate level of risk aversion in this sample.
- While we don’t have time to discuss this in detail, note that we incorporated a behavioural error term (μ) in the model.
- This is the “contextual utility” (CU) error specification of Wilcox (2011) which allows one to make robust inferences concerning the “stochastically more risk averse than” relation.
- This CU error specification is a particular operationalisation of the Fechner (1860) error term that will be used in the discounting models.

Risk preference results: Heterogenous preferences

- One can incorporate heterogeneity of preferences in this model by including a set of demographic covariates, as explained earlier.
- In effect, one conditions the risk preference estimates on observable characteristics of the sample, which then allows one to focus on the distribution of risk preferences in the sample – see Table II and Figure II below.

TABLE II: EXPECTED UTILITY THEORY

MLE ESTIMATES - HETEROGENOUS PRFERENCES & CU ERROR

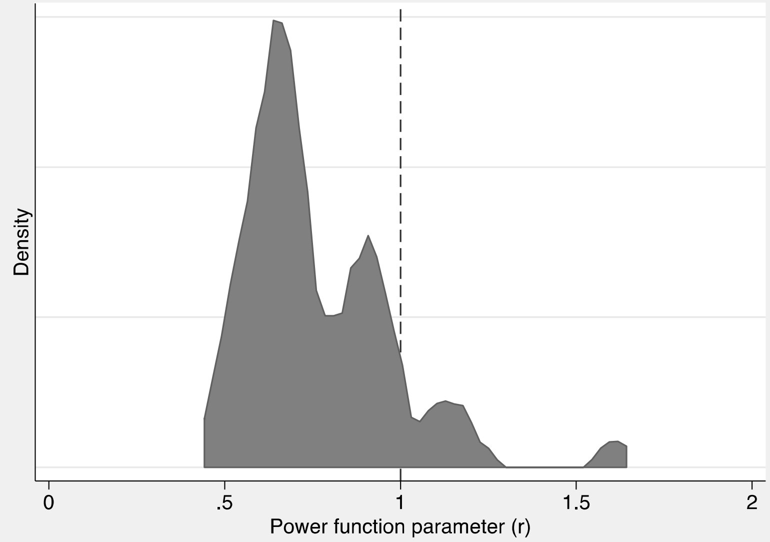
	Model	
	Estimate	Standard Error
Power function coefficient (r)		
Age	-0.063**	(0.031)
Male	-0.118	(0.249)
Black	0.113	(0.179)
White	0.38	(0.237)
Smoke	0.041***	(0.011)
Constant	-0.619	(1.792)
Error term (μ)		
Decision time	0.001	(0.002)
Constant	0.159***	(0.014)
N	2580	
log-likelihood	-1159.958	

Results account for clustering at the individual level

Standard errors in parentheses

* p<0.10, ** p<0.05, *** p<0.01

Figure II: EUT model - Distribution of the power function parameter



Risk preference results: Alternative theories of choice under risk

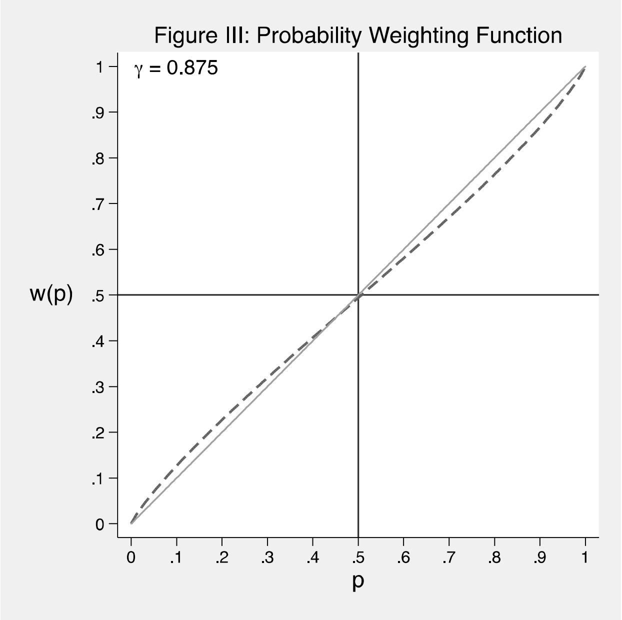
- In the previous slides we assumed that subjects made choices according to expected utility theory (EUT) which is one of the dominant theories of choice under risk.
- One of the great strengths of the statistical approach that we adopt is that it allows one to estimate the parameters of other theories of choice under risk with minimal fuss.
- Prospect theory and rank-dependent expected utility (RDEU) theory are two of the more popular alternatives to EUT and one can simply re-write the likelihood function from earlier to estimate the parameters of interest in these models.
- As a simple example we present the estimates of a rank-dependent expected utility model on the next slide.
- Note that under RDEU, risk aversion is generated both by the shape of the utility function and the shape of the so-called probability weighting function which reflects probability pessimism or optimism.

Risk preference results: RDEU with homogenous preferences

TABLE III: RANK-DEPENDENT EXPECTED UTILITY THEORY
MLE ESTIMATES - HOMOGENOUS PREFERENCES

	Model
	CU error Normal CDF
Power function coefficient (r)	0.700***
	(0.082)
Probability Weighting Function coefficient (γ)	0.875***
	(0.046)
Error (μ)	0.166***
	(0.013)
N	2640
log-likelihood	-1224.053

Results account for clustering at the individual level
Standard errors in parentheses
* p<0.10, ** p<0.05, *** p<0.01



- The estimate of *r* = 0.700 is similar to the EUT model and implies a moderate level of risk aversion which is attributable to utility function curvature.
- Interestingly, the estimate of γ = 0.875 implies overweighting of low probabilities (probability optimism) and underweighting of high probabilities (probability pessimism) as is evident in Figure III.
- This result mirrors those found by Kahneman & Tversky (1979) and Tversky and Kahneman (1992), amongst others.

Time preference results

- We now turn to some time preference results.
- As discussed previously, a subject is indifferent between two income options x_t and x_{t+T} if and only if:

  $$U(x_t) = (1/(1+\delta)^T)\, U(x_{t+T})$$

- Note that this indifference condition relies on estimates of the Power utility function U(•) introduced earlier.
- What this means is that one cannot infer the level of the individual discount rate without knowing or assuming something about a person's utility function.
- To illustrate this point we first present estimates of the sample's discount rate assuming risk neutrality (i.e. a linear utility function).
- We then show the effect on these estimates when one jointly estimates the utility function parameter *r* and the discount rate δ (i.e. we allow for risk aversion).

Time preference results: Linear utility function

ACTUARIAL SOCIETY OF SOUTH AFRICA

TABLE IV: EXPONENTIAL & HYPERBOLIC DISCOUNTING
MLE ESTIMATES - RISK NEUTRAL UTILITY FUNCTION

	Model
	Exponential Fechner error
Discount rate (δ)	3.239***
	(0.346)
Error (ν)	24.918***
	(2.176)
N	8340
log-likelihood	-4339.051

Results account for clustering at the individual level
Standard errors in parentheses
* p<0.10, ** p<0.05, *** p<0.01

- As is evident, the estimate of δ = 3.239 assuming a linear utility function implies a remarkably high discount rate (> 300%)
- This result is similar to many in the early literature on discount rate experiments and is an artifact of the assumption of risk neutrality.
- Note that as in our risk preference results we include a so-called Fechner error term to allow for subject errors from the perspective of the deterministic exponential discounting model.

Time preference results: Concave utility function

ACTUARIAL SOCIETY OF SOUTH AFRICA

TABLE V: EXPONENTIAL & HYPERBOLIC DISCOUNTING
MLE ESTIMATES - POWER UTILITY FUNCTION

	Model
	EUT Exponential
Power function coefficient (r)	0.289***
	(0.037)
Discount rate (δ)	0.519***
	(0.084)
Risk error (μ)	0.187***
	(0.013)
Time error (ν)	0.172***
	(0.059)
N	13900
log-likelihood	-7619.396

Results account for clustering at the individual level
Standard errors in parentheses
* p<0.10, ** p<0.05, *** p<0.01

- Table V displays estimates of the discount rate δ when it is estimated jointly with the Power function coefficient *r*.
- As is evident, the estimate of the discount rate δ = 0.519 is now significantly less than it was when we adopted the assumption of risk neutrality.
- This highlights the importance of joint estimation for drawing robust inferences from time discounting data.

Time preference results: Hyperbolic discounting

- As we discussed in the previous section on risk preferences, it is a straightforward exercise to estimate a hyperbolic discounting model as opposed to an exponential discounting model by altering the likelihood function in appropriate ways.
- In the table below, we show the estimates from a model of hyperbolic discounting which allows for curvature of the utility function.

TABLE VI: EXPONENTIAL & HYPERBOLIC DISCOUNTING
MLE ESTIMATES - POWER UTILITY FUNCTION

	Model
	EUT Hyperbolic
Power function coefficient (r)	0.315***
	(0.032)
Discount rate (δ)	0.483***
	(0.058)
Risk error (μ)	0.177***
	(0.010)
Time error (ν)	0.215***
	(0.063)
N	13900
log-likelihood	-7594.206

Results account for clustering at the individual level
Standard errors in parentheses
* p<0.10, ** p<0.05, *** p<0.01

Time preference results: A mixture of exponential and hyperbolic discounting

- Given the relative ease in estimating different models, a question which naturally arises is: which model should be estimated to best explain the data?
- While one can conduct statistical tests to determine which model, in and of itself, best characterises the data, we prefer a different approach.
- Rather than estimate an exponential discounting model and a hyperbolic discounting model separately, one can estimate a mixture of these two models and allow the data to tell us what proportion of choices is best explained by each model.
- To do so we specify a ‘grand likelihood’ function which is just a probability-weighted average of the likelihoods of the two discounting models.

Time preference results: A mixture of exponential and hyperbolic discounting

ACTUARIAL SOCIETY OF SOUTH AFRICA

- If we let π^E represent the probability that the exponential discounting model is correct, and $\pi^H = (1 - \pi^E)$ denote the probability that the hyperbolic discounting model is correct, then the grand likelihood is the probability-weighted average of the two conditional likelihoods L^E and L^H for the exponential and hyperbolic models, respectively. Thus, the likelihood for our mixture model is defined by

$$\ln L(\pi^E, \delta; y, \mathbf{X}) = \sum \ln[(\pi^E \times L^E) + (\pi^H \times L^H)]$$

- Note that this likelihood is maximised to estimate the parameters of each model and the weight accorded to each model in the data.
- The following slide presents the estimates from a mixture model of exponential and hyperbolic discounting which is estimated jointly with the curvature of the utility function and which takes into account the potential for errors by subjects.

Time preference results: A mixture of exponential and hyperbolic discounting

TABLE VII: MIXTURE MODEL DISCOUNTING
MLE ESTIMATES - POWER UTILITY FUNCTION

	Estimate	Std Error	*p*-value	95% Confidence Interval	
			Expected Utility Theory		
Power function coefficient (r)	0.303	0.034	0.000	0.236	0.370
			Exponential Discounting Model		
Mixture probability (π^E)	0.655	0.045	0.000	0.566	0.743
Discount rate (δ)	0.848	0.141	0.000	0.571	1.123
			Hyperbolic Discounting Model		
Mixture probability (π^H)	0.345	0.045	0.000	0.257	0.434
Discount rate (δ)	0.115	0.020	0.000	0.075	0.155
			Error Terms		
Risk Error (μ)	0.108	0.007	0.000	0.095	0.121
Time Error (ν)	0.023	0.009	0.001	0.006	0.039

H_0: $\pi^E = 0.5$, *p*-value = 0.001

- The exponential discounting model accounts for roughly 65.5% of the data while the hyperbolic discounting model accounts for roughly 34.5% of the data.
- The null hypothesis that both models explain 50% of the data is rejected at the 1% level.

Conclusion

- These rich quantitative estimates of the distributions of risk and time preferences in specific populations is a key input to predicting demand for insurance among people in non-traditional markets. (As noted previously, it is also important to gather qualitative information about portfolios of informal risk management mechanisms, e.g rights to draw from savings pools maintained by 'sisterhoods', that poor people hold.)

- Experience in other developing countries shows that, where risk and time preferences are measured accurately, it is possible to design microinsurance products that attract take-up rates on which providers can make profits.